FLOOD ROUTING IN MEANDERING RIVERS WITH FLOOD PLAINS

by

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ABSTRACT

Unsteady flow in a natural river which meanders through a wide flood plain is complicated by large differences in hydraulic resistance and cross-sectional geometry of the river channel and the flood plain. The unsteady flow is further complicated by the tendency for a portion of the flow to "short-circuit" along the more direct route afforded by the flood plain rather than following the longer route along the meandering channel. Thus, the wave attenuation and the time of travel of the portion of the flow in the channel differs from that in the flood plain due to differences in the hydraulic properties and flow-path distances of the channel and flood plain.

A mathematical model for routing floods in meandering rivers with flood plains is presented. The technique is based on a modified form of the complete one-dimensional equations of unsteady flow and thus avoids the obvious use of the more complex and computationally time consuming two-dimensional equations. The one-dimensional equations are modified such that the flow in the meandering channel and flood plain are identified separately. Thus, the differences in both hydraulic properties and flow-path distance are taken into account in a physically meaningful way, but one that is one-dimensional in concept. This development differs from conventional one-dimensional treatment of unsteady flows in rivers with flood plains wherein the flow is either averaged across the total cross-sectional area (channel and flood plain) or the flood plain is treated as off-channel storage, and the reach lengths of the channel and flood plain are assumed to be identical.

The modified equations contain the same two unknowns (discharge and water surface elevation) as the conventional equations; hence, the same numerical solution techniques applicable to the conventional one-dimensional unsteady flow equations may be used. In this paper, a weighted four-point implicit finite difference technique is used for reasons of its versatility and computing efficiency.

The mathematical model is compared with two conventional flood-plain routing models and found to produce appropriately smaller wave attenuation and travel time, especially when channel meander is a factor. The mathematical model is used to simulate a number of hydrographs for varying flood plain to channel ratios of flow-path distance, roughness, and width in order to determine both qualitatively and quantitatively the modifying effects of flood plains on floods propagating through meandering rivers. Wave attenuation and travel time are found to increase as flood-plain roughness and width increase and as the extent of channel meander decreases.

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INTRODUCTION

Unsteady flow in a natural river which meanders through a wide flood plain is complicated by large differences in geometric and hydraulic characteristics between the river channel and the flood plain, as well as the extreme differences in the hydraulic roughness coefficient. The flow is further complicated by the meandering of the main channel within the flood plain, which causes a portion of the total flow to "short-circuit" and proceed downstream along the more direct course afforded by the flood plain rather than along the more circuitous route of the meandering channel. This tendency for short-circuiting of the flow is enhanced by the greater longitudinal slope associated with the flood plain than that of the main channel; however, the short-circuiting effect is diminished by the greater hydraulic roughness of the flood plain. Further complexities are created by portions of the flood plain which act as dead storage areas, wherein the flow velocity is negligible. Another flow complexity occurs due to the interaction of the flows in the main channel and the flood plain; the direction of the lateral exchange of flow between the two watercourses depends on whether the flood wave is rising or receding, which, in turn, affects the magnitude of the associated energy loss.

In the literature [Liggett and Cunge, 1975; Miller and Cunge, 1975], it is evident that the one-dimensional mathematical models proposed for simulating unsteady flows in natural rivers have, for the most part, ignored the above flow complexities. Most either treat the flood plain or some portion of it as off-channel dead storage, or the main river channel and the flood plain are lumped together to form a composite channel in which the significantly different particle velocities and wave speeds of the flows in the main channel and in the flood plain are averaged together. Each of these techniques provides only a rough approximation of the actual flow, while the problem of short-circuiting is usually ignored; although, Perkins [1970] approached the problem by linearly adjusting the channel flow-path length as the flood plain is inundated.

The purpose of this paper is to present a one-dimensional mathematical model for simulating unsteady flows in a meandering river within a wide flood plain. Although the proposed model is also an approximation of the complex flow in such a natural watercourse, it does directly consider the influence of the unequal flow velocities and different degrees of roughness in the main river channel and the flood plain, the influence of different lengths and slopes of the flow paths of the channel and the flood plain, and the influence of dead storage areas. The energy loss due to the interaction of channel and flood-plain flows, as well as the problems of simulating helicoidal flow at river bends, traveling eddies, etc., which are associated with natural river channels are not considered in the proposed model.

MATHEMATICAL MODEL

Governing Equations

The basic concept of the model is to treat the flows in the river channel and the flood plain separately and from a one-dimensional point of view [Fread, 1975]. Thus, the model is based on a modified form of the one-dimensional equations of unsteady flow and thereby avoids the obvious use of the more complex and computationally time-consuming two-dimensional equations. This approach is warranted since the purpose of the model is to route floods and thus determine the celerity and transformation of the flood wave as it proceeds downstream through the river channel and the flood plain. These characteristics of the flood wave are influenced predominantly by the one-dimensional motion of the flood wave along the longitudinal axes of the river and the flood plain.

In order to treat the flows in the channel and flood plain separately, it is important that the geometric, roughness, and flow-path characteristics of both the river channel and flood plain be preserved in the governing one-dimensional equations. Using a subscript "c" to denote variables pertaining to the river channel, the complete one-dimensional equations of unsteady flow in a prismatic or non-prismatic river channel of arbitrary cross-sectional shape [Liggett, 1975; Stoker, 1957] are:

$$\frac{\partial A_c}{\partial t} + \frac{\partial Q_c}{\partial x_c} = 0 \tag{1}$$

$$\frac{\partial Q_c}{\partial t} + \frac{\partial (Q_c^2/A_c)}{\partial x_c} + gA_c\left(\frac{\partial h_c}{\partial x_c} + S_c\right) = 0 \tag{2}$$

Likewise, using a subscript "f" to denote variables pertaining to the flood plain, the one-dimensional equations of unsteady flow can be written for the flood-plain flow as follows:

$$\frac{\partial A_f}{\partial t} + \frac{\partial A_s}{\partial t} + \frac{\partial Q_f}{\partial x_f} - q = 0 \tag{3}$$

$$\frac{\partial Q_f}{\partial t} + \frac{\partial (Q_f^2/A_f)}{\partial x_f} + gA_f\left(\frac{\partial h_f}{\partial x_f} + S_f\right) - qv_{1x} = 0 \tag{4}$$

The variables in Eqs. 1-4 are defined as follows: Q = the discharge, L^3/T; A = the cross-sectional area, L^2; A_s = the off-channel dead storage area, L^2; h = the water surface elevation, L; S = the friction slope, L/L; q = the lateral inflow, L^2/T; v_{1x} = the velocity of the lateral inflow in the direction of x-axis of the flood plain, L/T; x = distance along the longitudinal axes of the channel or flood plain, L; t = time, T; and g = the acceleration due to gravity, L/T^2.

Upon adding the flows in the channel and the flood plain, the equations of unsteady flow for the combined flow become:

$$\frac{\partial(A_c + A_f + A_s)}{\partial t} + \frac{\partial Q_c}{\partial x_c} + \frac{\partial Q_f}{\partial x_f} - q = 0 \tag{5}$$

$$\frac{\partial(Q_c + Q_f)}{\partial t} + \frac{\partial(Q_c^2/A_c)}{\partial x_c} + \frac{\partial(Q_f^2/A_f)}{\partial x_f} + gA_c\left(\frac{\partial h_c}{\partial x_c} + S_c\right) + gA_f\left(\frac{\partial h_f}{\partial x_f} + S_f\right) - qv_{1x} = 0 \tag{6}$$

Eqs. 5-6 contain only four unknowns, Q_c, Q_f, h_c, and h_f because A can be expressed as a function of h, S can be expressed as a function of Q and h, and q, v_{1x}, g, x, t are known quantities. Since there are only two equations, it is desirable to reduce the number of unknowns to two.

Following a one-dimensional approach, the water surface is assumed to be horizontal across the river channel and the flood plain; therefore:

$$h_c = h_f = h \tag{7}$$

Thus, h_c and h_f can be replaced by h in Eqs. 5-6, thereby eliminating one of the four unknowns.

It is further assumed that the friction slope in the channel and in the flood plain can be expressed by Manning's equation, in which the slope S is approximated as:

$$S \simeq \Delta h/\Delta x \tag{8}$$

This approximation neglects the contribution of inertia effects in evaluating the friction loss, yet is reasonable in the case of slowly varying transients such as flood waves moving through meandering rivers.

An approximate ratio of the flow in the flood plain to that in the river channel can be found using Manning's equation with S approximated by Eq. 8. Thus

$$\frac{Q_f}{Q_c} = \frac{\dfrac{1.486}{n_f} A_f R_f^{2/3} \left(\dfrac{\Delta h}{\Delta x_f}\right)^{1/2}}{\dfrac{1.486}{n_c} A_c R_c^{2/3} \left(\dfrac{\Delta h}{\Delta x_c}\right)^{1/2}} \tag{9}$$

in which n = the Manning's roughness coefficient, $T/L^{1/3}$; R = the hydraulic radius, L, which is approximated herein by A/B, where B is the top width of the water surface within the cross section, L; $\Delta h/\Delta x$ = the change in water surface elevation per incremental distance along the channel or flood plain axis, L/L.

The above flow ratio is defined as:

$$\psi = \frac{Q_f}{Q_c} = \frac{n_c A_f}{n_f A_c}\left(\frac{R_f}{R_c}\right)^{2/3}\left(\frac{\Delta x_c}{\Delta x_f}\right)^{1/2} \tag{10}$$

The total flow in the channel and flood plain is the sum of two separate flows, i.e.,

$$Q = Q_c + Q_f \tag{11}$$

From Eq. 10, it is seen that

$$Q_f = \psi Q_c \tag{12}$$

Then, substituting Eq. 12 in Eq. 11, the following is obtained:

$$Q_c = \Phi Q \tag{13}$$

where

$$\Phi = \frac{1}{1+\psi} \tag{14}$$

Likewise, using Eqs. 11-12, the following can be obtained also:

$$Q_f = TQ \tag{15}$$

where

$$T = \frac{\psi}{1+\psi} \tag{16}$$

Since Φ and T are both functions of ψ, which, in Eq. 10, is seen to be a function of only one unknown variable (h), Eqs. 13 and 15 can be used to express Q_c and Q_f in terms of Q and h. Thus, upon substituting Eqs. 7, 11, 13, and 15 in Eqs. 5-6, only two unknowns, Q (total discharge) and h (water surface elevation), remain, i.e.,

$$\frac{\partial A}{\partial t} + \frac{\partial(\Phi Q)}{\partial x_c} + \frac{\partial(TQ)}{\partial x_f} - q = 0 \tag{17}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial(\Phi^2 Q^2/A_c)}{\partial x_c} + \frac{\partial(T^2 Q^2/A_f)}{\partial x_f} + gA_c\left(\frac{\partial h}{\partial x_c} + S_c\right) + gA_f\left(\frac{\partial h}{\partial x_f} + S_f\right) - qv_{1x} = 0 \tag{18}$$

in which

$$A = A_c + A_f + A_s \tag{19}$$

Eqs. 17-18 are the governing differential equations of one-dimensional flow in a natural meandering river with a flood plain. Eq. 17 conserves the mass of the flow and Eq. 18 conserves the momentum.

Eq. 18 neglects momentum considerations for the lateral exchange of flow between the channel and the flood plain. The exchange is assumed to be completely described by the equation of continuity (Eq. 17).

Eqs. 17-18 constitute a system of partial differential equations of the hyperbolic type. They contain two independent variables, x and t, and two dependent variables, h and Q; the remaining terms are either functions of x, t, h, and/or Q, or they are constants. These equations are not amenable to analytical solutions except in cases where the channel geometry and boundary conditions are uncomplicated and the non-linear properties of the equations are either neglected or made linear. The equations may be solved numerically by performing two basic steps. First, the partial differential equations are represented by a corresponding set of finite difference algebraic equations; and second, the system of algebraic equations is solved in conformance with prescribed initial and boundary conditions.

Numerical Solution

Eqs. 17-18 are modified forms of the conventional one-dimensional equations of unsteady flow (Eqs. 1-2). They contain the same two unknowns (water surface elevation and discharge) as the conventional equations; hence, the same numerical solution techniques applicable to the conventional equations may be used. Accordingly, Eqs. 17-18 can be solved by either explicit or implicit finite difference techniques [Liggett and Cunge, 1975]. Explicit methods, although simpler in application, are not suitable for the simulation of long-term unsteady flow phenomena, such as flood waves in rivers, because they are restricted by mathematical stability considerations to very small computational time steps (on the order of a few minutes). Such small time steps cause the explicit methods to be very inefficient in the use of computer time. Implicit finite difference techniques, however, have no restrictions on the size of the time step due to mathematical stability; however, convergence considerations may require its size to be limited to something less than a few hundred times that of the explicit method, depending on the hydraulic properties of the unsteady flow and the size of the distance step.

Of the various implicit schemes that have been developed, the "weighted four-point" scheme first used by Preissman [1961] and recently by Quinn and Wylie [1973], Chaudry and Contractor [1973] and Fread [1974b] appears most advantageous since it can readily be used with unequal distance steps and its stability-convergence properties can be controlled easily. In the weighted four-point implicit finite difference scheme, the continuous x-t region in which solutions of h and Q are sought is represented by a rectangular net of discrete points. The net points are determined by the intersection of lines drawn parallel to the x and t axes. Those

parallel to the x axis represent time lines; they have a spacing of Δt, which need not be constant. Those parallel to the t axis represent discrete locations or nodes along the river (x axis); they have a spacing of Δx, which also need not be constant. Each point in the rectangular network can be identified by a subscript (i) which designates the x position and a superscript (j) which designates the time line.

The time derivatives are approximated by a forward difference quotient centered between the i^{th} and i+1 points along the x axis, i.e.,

$$\frac{\partial K}{\partial t} \simeq \frac{K_i^{j+1} + K_{i+1}^{j+1} - K_i^j - K_{i+1}^j}{2\,\Delta t} \tag{20}$$

where K represents any variable.

The spatial derivatives are approximated by a forward difference quotient positioned between two adjacent time lines according to weighting factors of θ and $1-\theta$, i.e.,

$$\frac{\partial K}{\partial x} \simeq \theta\left(\frac{K_{i+1}^{j+1} - K_i^{j+1}}{\Delta x}\right) + (1-\theta)\left(\frac{K_{i+1}^j - K_i^j}{\Delta x}\right) \tag{21}$$

Variables other than derivatives are approximated at the time level where the spatial derivatives are evaluated by using the same weighting factors, i.e.,

$$K \simeq \theta\left(\frac{K_i^{j+1} + K_{i+1}^{j+1}}{2}\right) + (1-\theta)\left(\frac{K_i^j + K_{i+1}^j}{2}\right) \tag{22}$$

A θ weighting factor of 1.0 yields the fully implicit or backward difference scheme used by Baltzer and Lai [1968], Dronkers [1969]. A weighting factor of 0.5 yields the box scheme used by Amein and Fang [1970], Contractor and Wiggert [1971]. The influence of the θ weighting factor on the accuracy of the computations was examined by Fread [1974a], who concluded that the accuracy decreases as θ departs from 0.5 and approaches 1.0. This effect becomes more pronounced as the magnitude of the computational time step increases. In this paper, a weighting factor of 0.55 is used so as to minimize the loss of accuracy associated with greater values while avoiding the possibility of a weak or pseudo instability noticed by Baltzer and Lai [1968], Quinn and Wylie [1972], Chaudhry and Contractor [1973], and Fread [1975] when a θ of 0.5 is used.

When the finite difference operators defined by Eqs. 20-22 are used to replace the derivatives and other variables in Eqs. 17-18, the following weighted four-point implicit difference equations are obtained:

$$\frac{A_i^{j+1} + A_{i+1}^{j+1} - A_i^j - A_{i+1}^j}{2\,\Delta t_j} + \theta\left(\frac{(\Phi Q)_{i+1}^{j+1} - (\Phi Q)_i^{j+1}}{\Delta x_{c_i}} - q_{i+1/2}^{j+1}\right)$$

$$+ (1-\theta)\left(\frac{(TQ)_{i+1}^{j} - (TQ)_i^{j}}{\Delta x_{f_i}} - q_{i+1/2}^{j}\right) = 0 \tag{23}$$

$$\frac{Q_i^{j+1} + Q_{i+1}^{j+1} - Q_i^j - Q_{i+1}^j}{2\,\Delta t_j} + \theta\left[\frac{(\Phi^2Q^2/A_c)_{i+1}^{j+1} - (\Phi^2Q^2/A_c)_i^{j+1}}{\Delta x_{c_i}}\right.$$

$$+ \frac{(T^2Q^2/A_f)_{i+1}^{j+1} - (T^2Q^2/A_f)_i^{j+1}}{\Delta x_{c_i}} + g\bar{A}_c^{j+1}\left(\frac{h_{i+1}^{j+1} - h_i^{j+1}}{\Delta x_{c_i}} + \bar{S}_c^{j+1}\right)$$

$$\left. + g\bar{A}_f^{j+1}\left(\frac{h_{i+1}^{j+1} - h_i^{j+1}}{\Delta x_{f_i}} + \bar{S}_f^{j+1}\right) - (qv_{1x})_{i+1/2}^{j+1}\right]$$

$$+ (1-\theta)\left[\frac{(\Phi^2Q^2/A_c)_{i+1}^{j} - (\Phi^2Q^2/A_c)_i^{j}}{\Delta x_{c_i}} + \frac{(T^2Q^2/A_f)_{i+1}^{j} - (T^2Q^2/A_f)_i^{j}}{\Delta x_{f_i}}\right.$$

$$\left. + g\bar{A}_c^{j}\left(\frac{h_{i+1}^{j} - h_i^{j}}{\Delta x_{c_i}} + \bar{S}_c^{j}\right) + g\bar{A}_f^{j}\left(\frac{h_{i+1}^{j} - h_i^{j}}{\Delta x_{f_i}} + \bar{S}_f^{j}\right) - (qv_{1x})_{i+1/2}^{j}\right] = 0 \tag{24}$$

where

$$\bar{S}_f = \frac{n\,\bar{Q}|\bar{Q}|}{2.21\,\bar{A}^2\bar{R}^{4/3}} \tag{25}$$

in which,

$$\bar{Q} = 0.5\ (Q_i + Q_{i+1}) \tag{26}$$

$$\bar{A} = 0.5\ (A_i + A_{i+1}) \tag{27}$$

$$\bar{R} = 0.5\ (R_i + R_{i+1}) \tag{28}$$

Eqs. 23-24 constitute a system of algebraic equations that are non-linear with respect to the unknowns, i.e., the values of the dependent variables h and Q at the net points i and i+1 at the time line designated as j+1.

The terms associated with the j^{th} time line are known from either the initial conditions or previous computations. The initial conditions refer to values of h and Q at each node along the x axis for the first time line (j=1). They are obtained from a previous unsteady flow solution, or they can be estimated since small errors in the initial conditions dampen out within a few time steps.

Eqs. 23-24 cannot be solved in an explicit or direct manner for the unknowns since there are four unknowns and only two equations. However, if Eqs. 23-24 are applied to each of the (N-1) rectangular grids between the upstream and downstream boundaries, a total of (2N-2) equations with 2N unknowns can be formulated. (N denotes the total number of nodes.) Then, prescribed boundary conditions, one at the upstream boundary and one at the downstream boundary, provide the necessary two additional equations required for the system to be determinate. The resulting system of 2N non-linear equations with 2N unknowns is solved by a functional iterative procedure, the Newton-Raphson method [Amein and Fang, 1970].

Computations for the iterative solution of the non-linear system are begun by assigning trial values to the 2N unknowns. Substitution of the trial values into the system of non-linear equations yields a set of 2N residuals. The Newton-Raphson method provides a means for correcting the trial values until the residuals are reduced to a suitable tolerance level. This is usually accomplished in one or two iterations through use of linear or parabolic extrapolation for the first trial values. If the Newton-Raphson corrections are applied only once, i.e., there is no iteration, the non-linear system of difference equations degenerates to the equivalent of a quasi-linear difference formulation which may require smaller time steps than the non-linear formulation for the same degree of numerical accuracy.

A system of 2N x 2N linear equations relates the corrections to the residuals and to a Jacobian coefficient matrix composed of partial derivatives of each equation with respect to each unknown variable in that equation. The coefficient matrix of the linear system has a banded structure which allows the system to be solved by a compact quad-diagonal Gaussian elimination algorithm [Fread, 1971], which is very efficient with respect to computing time and storage. The required storage is 2N x 4 and the required computational steps are approximately 38N.

The boundary conditions consist of a description of either water surface elevation (h) or discharge (Q) as a function of time at the upstream and downstream extremities of the study reach. The downstream boundary may also be a specified relationship between h and Q such as an empirical rating curve or normal stage-discharge relationship corrected for unsteady effects. For example, the upstream boundary could be given by the following:

$$Q_1^{j+1} - Q(t) = 0 \tag{29}$$

where Q(t) is the specified temporal variation of Q at the upstream boundary; and the downstream boundary could be given by the following stage-discharge relationship:

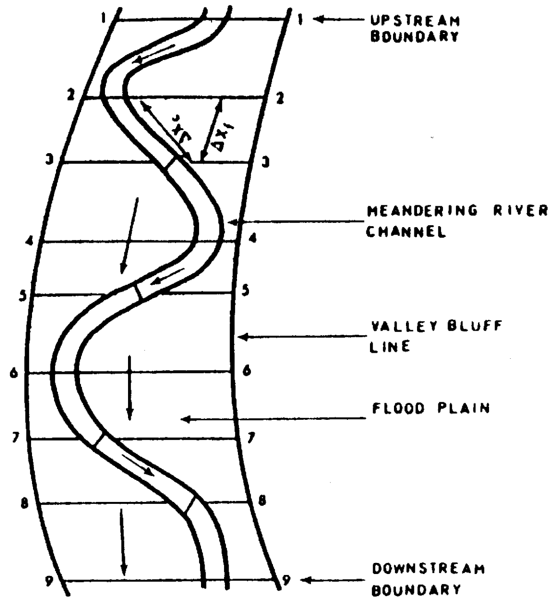


FIG. 1 DEFINITION PLAN OF IDEALIZED MEANDERING RIVER WITH FLOOD PLAIN

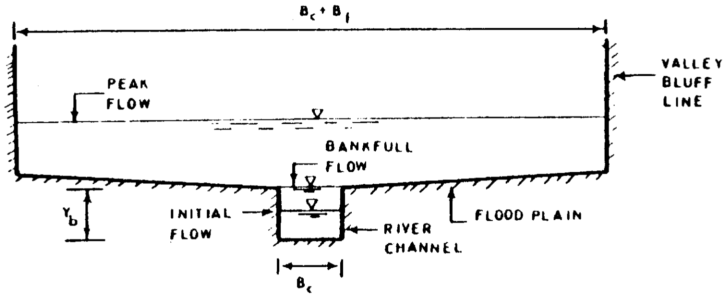


FIG. 2 CROSS-SECTION 5-5 OF IDEALIZED RIVER

$$Q_N^{j+1} - 1.49\left(\frac{AR^{2/3}S^{1/2}}{n}\right)_N^{j+1} = 0 \qquad (30)$$

where S is approximated by Eq. 8.

COMPARISON WITH CONVENTIONAL MODELS

The model developed herein is referred to as the "channel-flood plain" model to distinguish it from two other conventional models of rivers having flood plains with which it will be compared. The other models are denoted as the "off-channel storage" model and the "composite channel" model.

The off-channel storage model treats the flood plain as off-channel dead storage. In this model the effect of the flood plain on unsteady flows is assumed to be completely described by the continuity equation (Eq. 1) in which the term A_c is replaced by A, which is defined as:

$$A = A_c + A_f \qquad (31)$$

In this model, the velocity of the flow in the flood plain is assumed negligible and its momentum effects are not considered in Eq. 2.

The composite section model treats the channel and flood plain as one continuous cross-sectional area. In this model the term A_c in each of Eqs. 1-2 is replaced by A, as defined by Eq. 31. An equivalent Manning roughness coefficient (n_e), which is a weighted average of n_c and n_f, is used. It is based on the assumption that the total force resisting the flow is equal to the sum of the forces resisting the flow in the channel and in the flood plain. It is given by:

$$n_e = [(P_c n_c^2 + P_f n_f^2)/(P_c + P_f)]^{1/2} \qquad (32)$$

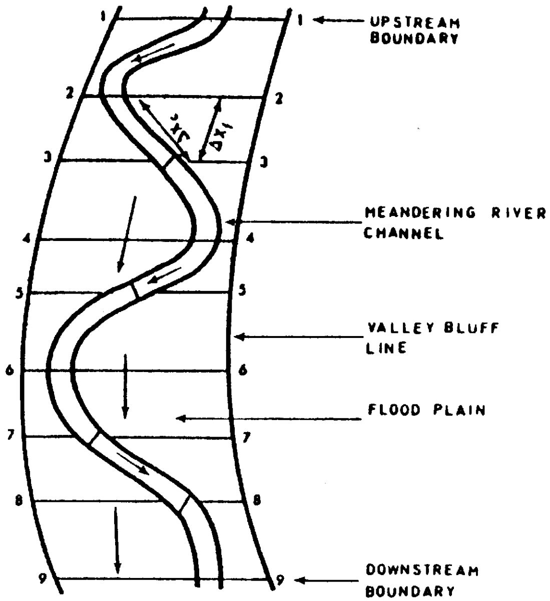
where P is the wetted perimeter of the cross-sectional area.

The same weighted four-point implicit finite difference method is used for the numerical solution procedure in the off-channel storage and composite channel models as was previously described for the channel-flood plain model.

An idealized meandering river with a significant flood plain (Fig. 1) and uniform cross section (Fig. 2) is used in the comparison of the simulation characteristics of the three models. The discharge-hydrograph is specified for the upstream end of a 100-mile reach of meandering river with a bottom slope of 1 foot/mile. The three models are used to compute the discharge hydrograph at the downstream boundary. In order to minimize numerical errors, 1-hour time steps and 1-mile distance steps are used. The extent of meander or sinousity is specified by the flow-path length ratio:

$$L_r = L_c/L_f \qquad (33)$$

where L_c is the length of the meandering channel and L_f is the length of the flood plain between the upstream and downstream boundaries. For the comparison



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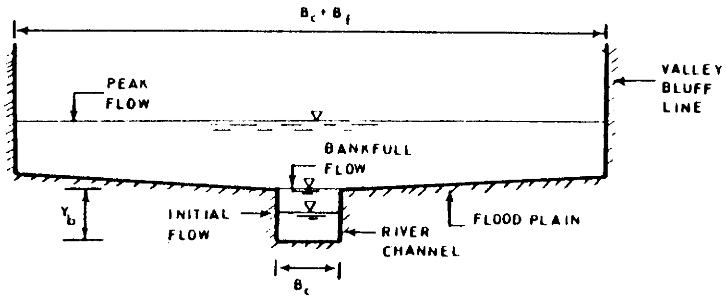

FIG. 2 CROSS-SECTION 5-5 OF IDEALIZED RIVER

$$Q_N^{j+1} - 1.49\left(\frac{AR^{2/3}S^{1/2}}{n}\right)_N^{j+1} = 0 \tag{30}$$

where S is approximated by Eq. 8.

COMPARISON WITH CONVENTIONAL MODELS

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$$n_e = [(P_c n_c^2 + P_f n_f^2)/(P_c + P_f)]^{1/2} \tag{32}$$

where P is the wetted perimeter of the cross-sectional area.

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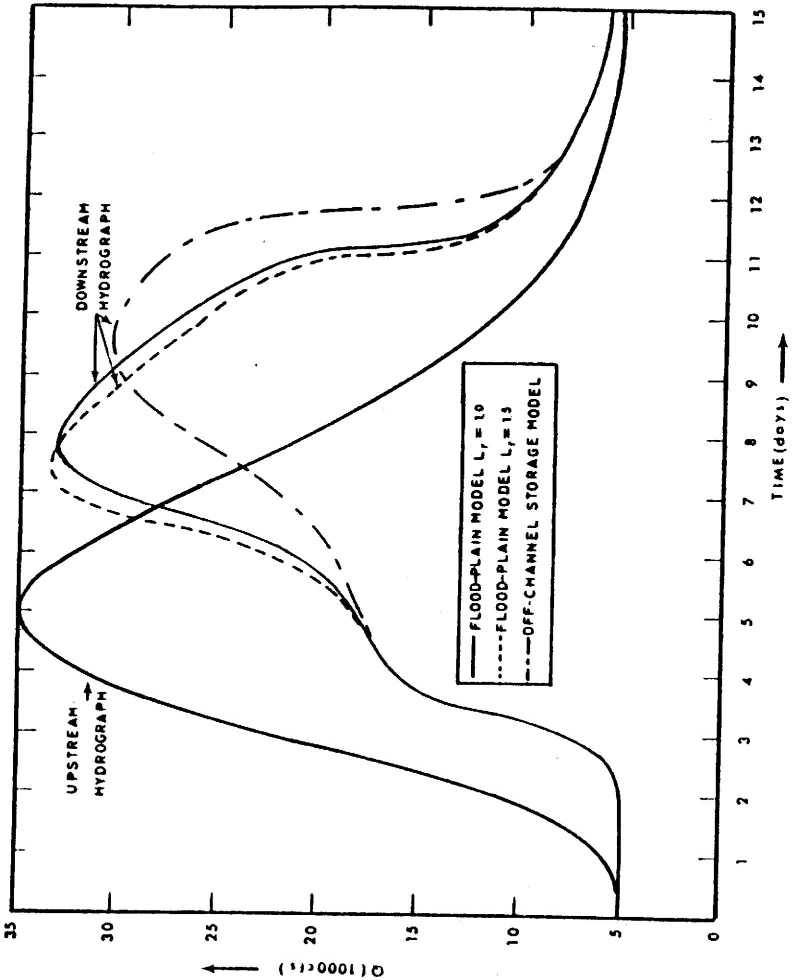

FIG. 3 TYPICAL HYDROGRAPHS OF CHANNEL-FLOOD PLAIN AND OFF-CHANNEL STORAGE MODELS

simulations, the pertinent characteristics of the channel and flood plain are: L_r = 1.0, 1.5, 2.0; n_c = 0.03; n_f = 0.06; B_c = 500 feet; B_f = 2,000 feet; Y_b = 10 feet; the flood plain slopes upward from the river to the valley bluff line at a rate of 1 foot per 1,000 feet; and there is no lateral inflow.

Typical computed hydrographs of the channel-flood plain model and the off-channel storage model are shown in Fig. 3. The composite model is not shown since computational problems are encountered when simulating the sudden change in top width (B) as the flow spills onto the wide flat flood plain.

For a critical assessment of the differences between the models, attention is focused on the attenuation (α_p) and travel time (τ_p) of the hydrograph peaks (Q_p). These are normalized to the attenuation (α_b) and travel time (τ_b) associated with the flow condition (Q_b) when the river channel is bankfull. Normalization about the bankfull condition focuses on the differences in the computed attenuation and travel time of each model due to the presence of the flood plain. Simulations are also obtained from the composite channel model with the flow simulation starting with the flood plain inundated sufficiently to eliminate the problem of large changes in the top width per change in flow depth as the flow spills onto the flood plain. These results can be compared with the other models since only the hydrograph peaks are of interest.

The attenuation characteristics of the three flood-plain models are shown in Fig. 4. The off-channel storage model attenuates the hydrograph much more than the other two models. The composite channel model is similar to the channel-flood plain model, particularly at the larger flows and when there is no channel meander (L_r=1.0). As the sinuosity of the river channel increases, the composite channel model deviates farther from the channel-flood plain model, the former attenuating the hydrograph more than the latter.

The travel time characteristics of the three models are shown in Fig. 5. Again, the off-channel storage model deviates significantly from the other two models as manifested by its considerably greater travel times. The composite channel model deviates more and more from the channel-flood plain model as the sinuosity (L_r) and the peak flow (Q_p) increase.

The channel-flood plain model produces results which approach those of the off-channel storage model as the flood-plain roughness and width increase and as the sinuosity decreases. Although the composite section model has the inherent problem of simulating the condition when the flow spills onto a wide flat flood plain, its simulation characteristics approach those of the channel-flood plain model as the difference in channel and flood-plain roughness decreases, as the sinuosity decreases, and as the flood-plain width decreases.

FLOOD WAVE MODIFICATIONS DUE TO FLOOD PLAINS

The channel-flood plain model was used to obtain a qualitative and quantitative description of the modifying effects that flood plains have on flood waves propagating through meandering river channels. Hydrographs having a wide range of peak values were routed through the idealized meandering river and flood plain (Figs. 1-2) described previously. The modifying effects of the flood plain on the attenuation and travel time of the hydrograph peaks were sought for varying channel and flood plain characteristics such as flow-path distance, hydraulic roughness, and width. The flood plain effects are expressed in terms of a normalized attenuation (α_p/α_b) and travel time (τ_p/τ_b) for varying discharges normalized to the bankfull flow, i.e., (Q_p/Q_b).

The extent of channel sinuosity (L_r) is an important factor in affecting the attenuation. This is evident in Fig. 6, where the attenuation decreases significantly as the sinuosity increases. The travel time (Fig. 7) also decreases as the channel meander increases. The decrease in attenuation and travel time results when a portion of the flow short-circuits along the shorter path afforded by the flood plain rather than following the longer circuitous channel. For a given sinuosity, attenuation tends to increase and travel time decrease as the flow increases; however, the trends are reversed when the flood plain flows and depths are small.

The relative roughness of the flood plain to the channel, i.e.,

$$n_r = n_f/n_c \qquad (34)$$

has an important effect on the attenuation and travel time characteristics. Attenuation (Fig. 8) and travel time (Fig. 9) increase significantly as the relative roughness of the flood plain to channel increases.

Similarly, the relative width of the flood plain to the channel, i.e.,

$$B_r = B_f/B_c \qquad (35)$$

affects the attenuation and travel time characteristics. Attenuation (Fig. 10) and travel time (Fig. 11) increase as the width ratio (B_r) increases.

In Figs. 8-11, the attenuation increases and travel time decreases as the flow increases, except at low flood plain flows where the trends are reversed.

SUMMARY

A mathematical model has been developed to route floods in natural meandering rivers with wide flood plains. The governing equations are modifications of the conventional one-dimensional equations of unsteady flow. The modified equations preserve the identity of the essential one-dimensional flow properties of the flow in the river channel and in the flood plain; also, the flow-path lengths and slopes of each watercourse are preserved, which allows the problem of flow short-circuiting to be realistically modeled. The modified equations contain the same two unknowns as the conventional equations; this allows the convenient application of conventional numerical solution techniques. An efficient and versatile weighted four-point implicit scheme has been applied to the governing equations.

The channel-flood plain model developed herein has been compared with two conventional methods of treating the flood plain. The channel-flood plain model provided results which differed from the other models, especially where channel sinuosity and associated flow short-circuiting are factors. The

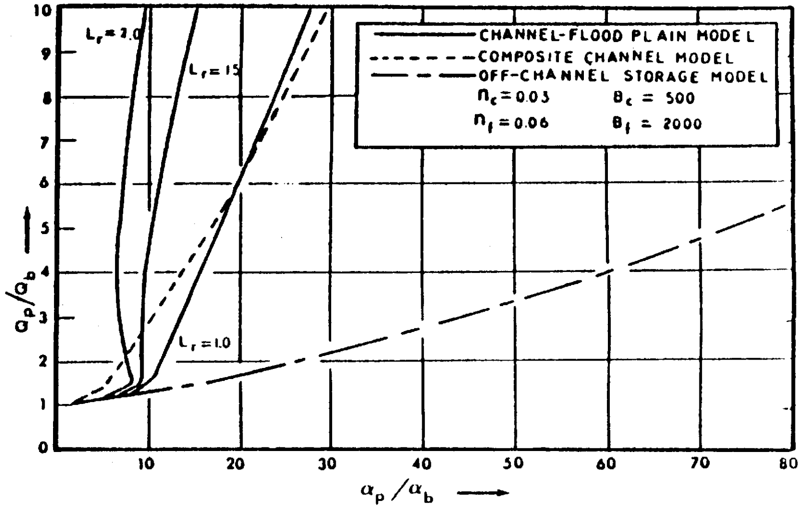

FIG. 4 ATTENUATION CHARACTERISTICS OF THREE FLOOD-PLAIN MODELS

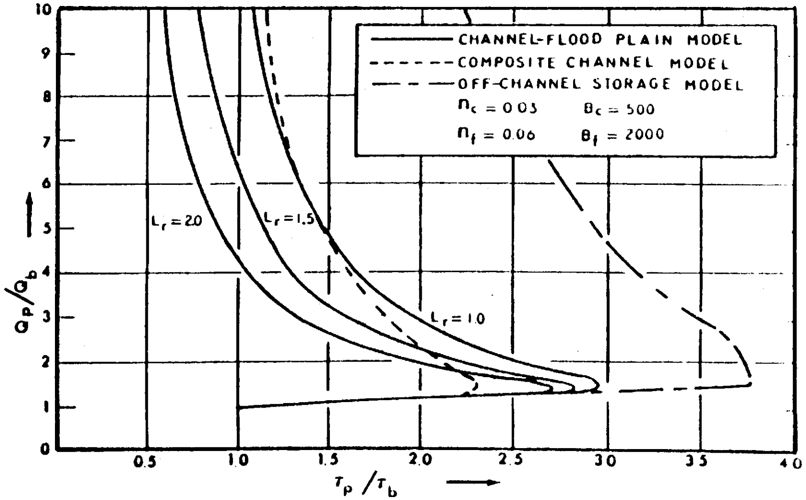

FIG. 5 TRAVEL TIME CHARACTERISTICS OF THREE FLOOD-PLAIN MODELS

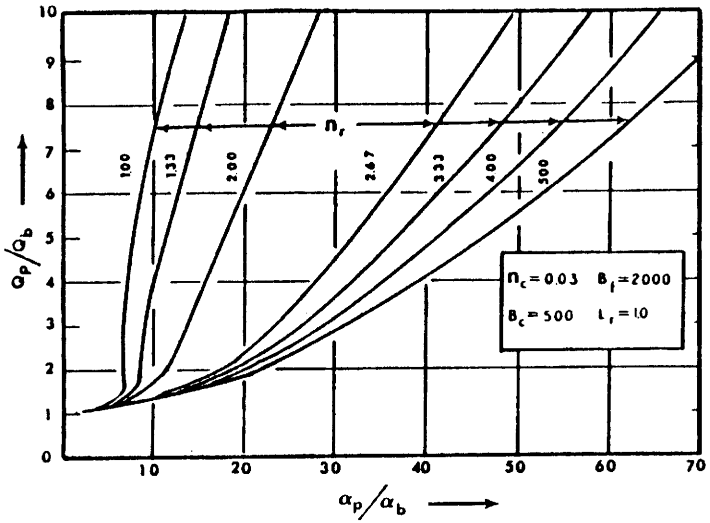

FIG. 8 EFFECT OF ROUGHNESS RATIO (n_r) ON FLOOD ATTENUATION

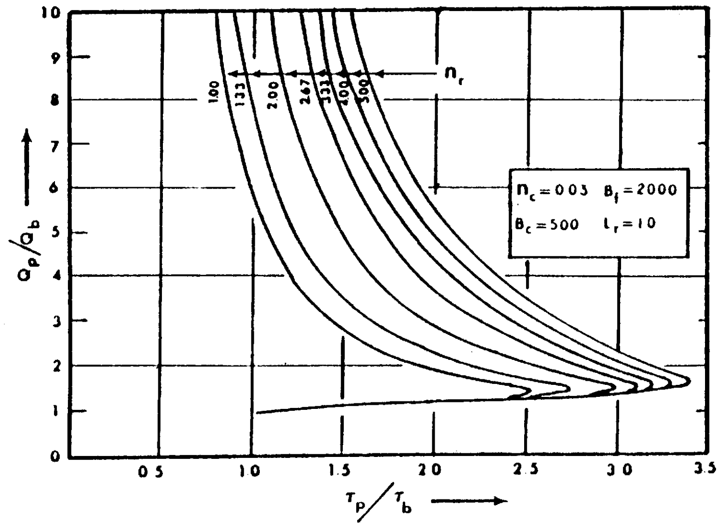

FIG. 9 EFFECT OF ROUGHNESS RATIO (n_r) ON FLOOD TRAVEL TIME

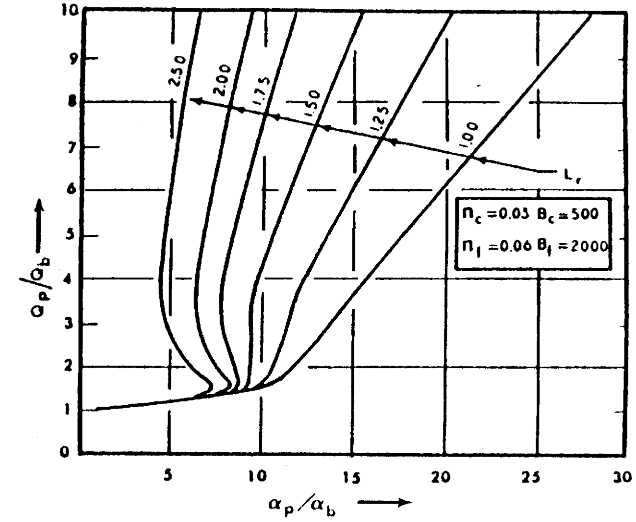

FIG. 6 EFFECT OF CHANNEL SINUOSITY (L_r) ON FLOOD ATTENUATION

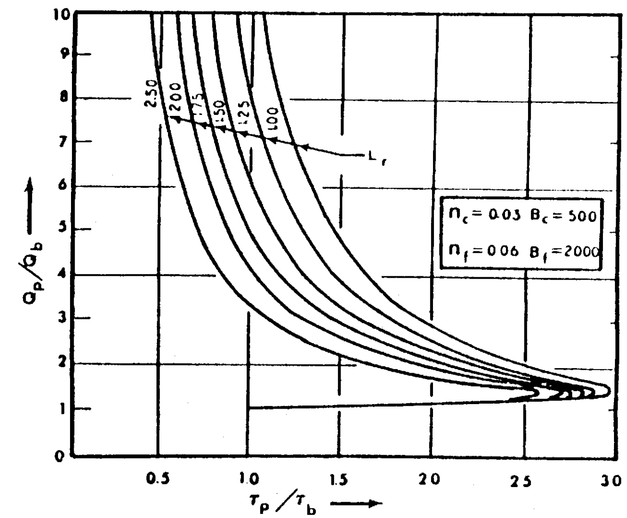

FIG. 7 EFFECT OF CHANNEL SINUOSITY (L_r) ON FLOOD TRAVEL TIME

computational problem associated with a composite channel model when the flow spills onto a wide flat flood plain is not encountered in the channel-flood plain model.

The channel-flood plain model has been used to simulate flows in an idealized meandering river with a flood plain. Flood peak attenuation and travel time were found to increase as flood-plain roughness and width increase and as channel sinuosity decreases. Attenuation increases and travel time decreases as the flood-plain flow increases except at low flood-plain flows when the trends are reversed.

REFERENCES

Amein, M., and C. S. Fang, Implicit flood routing in natural channels, J. Hydraul. Div. Amer. Soc. Civil Engr., 96 (HY12), 2481-2500, 1970.

Baltzer, R. A. and C. Lai, Computer simulation of unsteady flows in water-ways, J. Hydraul. Div. Amer. Soc. Civil Engr., 94 (HY4), 1083-1117, 1968.

Chaudhry, Y. M., and D. N. Contractor, Application of implicit method to surges in channels, Water Resour. Res., 9 (6), 1605-1612, 1973.

Contractor, D. N. and J. M. Wiggert, Numerical studies of unsteady flow in the James River, Bulletin 51, VPI-WRRC-Bull 51, Water Resour. Res. Ctr., Virginia Poly Inst. and State Univ., Blacksburg, Va., May 1972.

Dronkers, J. J., Tidal computations for rivers, coastal areas, and seas, J. Hydraul. Div. Amer. Soc. Civil Engr., 95 (HY1), 29-77, 1969.

Fread, D. L., Discussion of implicit flood routing in natural channels, M. Amein and C. S. Fang, J. Hydraul. Div. Amer. Soc. Civil Engr., 97 (HY7), 1156-1159, 1971.

Fread, D. L., Numerical properties of implicit four-point finite difference equations of unsteady flow, NOAA Tech. Memo NWS HYDRO-18, U.S. Dept. of Commerce, NOAA, National Weather Service, 1974a.

Fread, D. L., Implicit dynamic routing of floods and surges in the Lower Mississippi, presented at AGU Natl. Mtg., Wash., D.C., April 1974b.

Fread, D. L., Discussion of comparison of four numerical methods for flood routing, R. K. Price, J. Hydraul. Div. Amer. Soc. Civil Engr., 101 (HY3), 565-567, 1975.

Liggett, J. A., Basic equations of unsteady flow, Unsteady Flow in Open Channels, edited by K. Mahmood and V. Yevjevich, Vol. I, Chapter 3, 29-62, Water Resour. Pub., Ft. Collins, Colorado, 1975.

Liggett, J. A. and J. A. Cunge, Numerical methods of solution of the unsteady flow equations, Unsteady Flow in Open Channels, edited by K. Mahmood and V. Yevjevich, Vol. I, Chapter 4, 89-182, Water Resour. Pub., Ft. Collins, Colorado, 1975.

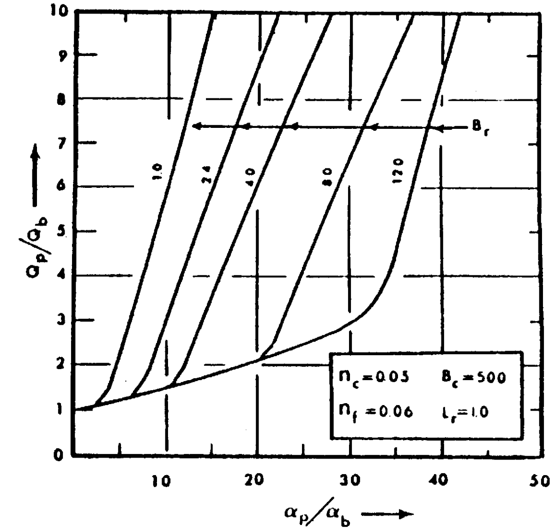

FIG. 10 EFFECT OF WIDTH RATIO (B_r) ON FLOOD ATTENUATION

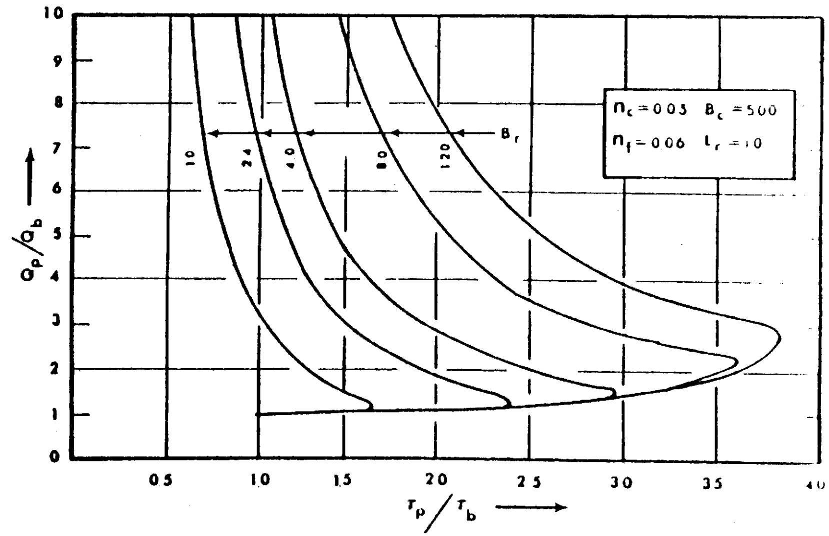

FIG. 11 EFFECT OF WIDTH RATIO (B_r) ON FLOOD TRAVEL TIME

Miller, W. A. and J. A. Cunge, Examples of one-dimensional flow modeling, Unsteady Flow in Open Channels, edited by K. Mahmood and V. Yevjevich, Vol. I, Chapter 11, 407-484, Water Resour. Pub., Ft. Collins, Colorado, 1975.

Perkins, F. E., Flood plain modeling, Water Resour. Bull., 6 (3), 375-383, 1970.

Preissmann, A., Propagation des intumescences dans les canaux et rivières, 1st Congrès de l'Assoc. Francaise de Calcul, Grenoble, 433-442, 1961.

Quinn, F. H. and E. B. Wylie, Transient analysis of the Detroit River by the implicit method, Water Resour. Res., 8 (6), 1461-1469, 1972.

Stoker, J. J., Water Waves, Interscience Pub., Inc., New York, 1957.